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Triple and quartic interactions of Higgs bosons in the two-Higgs-doublet model with CP-violation

M.N. Dubinin[1], A.V. Semenov[2]

[1] Institute of Nuclear Physics, Moscow State University, 119992 Moscow, Russia
[2] Laboratoire de Physique Theorique, LAPTH, Chemin de Bellevue, B.P. 110, 74941, Annecy-le-Vieux, Cedex, France

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Abstract. We consider the two-Higgs-doublet model with explicit CP-violation, where the effective Higgs potential is not CP-invariant at the tree level. The three neutral Higgs bosons of the model are the mixtures of CP-even and CP-odd bosons which exist in the CP-conserving limit of the theory. The mass spectrum and tree-level couplings of the neutral Higgs bosons to gauge bosons and fermions are significantly dependent on the parameters of the Higgs boson mixing matrix. We calculate the Higgs–gauge boson, Higgs–fermion, triple and quartic Higgs self-interactions in the MSSM with explicit CP-violation in the Higgs sector and CP-violating Yukawa interactions of the third generation scalar quarks. In some regions of the MSSM parameter space substantial changes of the self-interaction vertices take place, leading to significant suppression or enhancement of the multiple Higgs boson production cross sections.

1 Introduction

A general interest in the models with two (and more) Higgs doublets is maintained by the absence of a convincing argument in favor of only one generation of Higgs bosons when there are three known generations of fundamental fermions. Models with an extended Higgs sector provide richer physical possibilities than the standard scheme with one doublet. One of them is the possibility to introduce CP-violation beyond the Cabibbo–Kobayashi–Maskawa (CKM) mechanism, by means of the Higgs boson exchange amplitudes with complex Higgs boson–fermion vertices. Complex couplings can be generated either spontaneously [1], when the vacuum expectation values of the Higgs fields are complex and the couplings of the CP-invariant tree-level Higgs potential are real, or explicitly inserted [2] on the level of $SU(2) \times U(1)$-invariant potential terms, when the complex vacuum expectation values of the scalar fields correspond to the minimum of the hermitian potential with complex parameters, which is not CP-invariant (CP-invariance softly broken by the mass terms).

Various representations of the $SU(2) \times U(1)$-invariant two-doublet Higgs potentials have been considered in the literature. The two-doublet models with spontaneous CP-violation [1,3] make use of the potential of the general structure $-\mu^2\varphi^2 + \lambda\varphi^4$ without the dimension two μ_{12}^2-terms. Models with explicit CP-violation use either the potential with trivial minimization [2,4], or the potential with complex parameter μ_{12}^2 of the dimension two terms and complex parameters $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ in front of the dimension four potential terms [5,6], similar to the effective potential of the minimal supersymmetry (MSSM). The standard transformation (diagonalization) procedure from the level of primary fields which are the components of scalar doublets in $SU(2) \times U(1)$-invariant potential terms ($SU(2) \times U(1)$ eigenstates) to the physical fields (mass eigenstates) of the Higgs bosons should be consistently performed to respect the $SU(2) \times U(1)$ invariance and the minimization of the potential. We consider the diagonalization for the two different two-Higgs-doublet potential forms in great detail. A special case of the general two-Higgs-doublet model is represented by the Higgs sector of MSSM. Substantial radiative corrections to the Higgs boson masses and couplings are induced at the m_Z scale mainly by the third generation quarks t, b and the third generation scalar quarks [7]. In the special case of MSSM the multiparameter space of the general two-Higgs-doublet model is significantly reduced, providing possibilities of much less ambiguous phenomenological predictions.

Phenomenological consequences of the CP-violating Higgs–third generation squark Yukawa interactions in the Higgs–fermion and the Higgs–gauge boson sectors have been considered in [5]. We focus mainly on the self-interactions of the Higgs bosons. Experimental observation of the scalar boson signals should be followed by the verification of the Higgs mechanism as the essence of the gauge boson and fermion mass generation. Self-interactions of the Higgs fields lead to a non-trivial structure of the vacuum state with non-zero (and possibly complex) field tensions, initializing the spontaneous breakdown of $SU(2) \times U(1)$ symmetry. The reconstruction of the Higgs self-interaction potential from the data on multiple (mainly double and triple) Higgs boson production cross sections [8] requires the experimental measurements of triple and quartic Higgs boson self-interaction vertices, which is a nontrivial but valuable task for future high luminosity colliders, such as LHC and TESLA.

In Sect. 2 we discuss the diagonalization of the CP-invariant Higgs potential, represented in two different forms, in the general two-Higgs-doublet model (THDM), and we consider the special case of the Higgs sector in minimal supersymmetry (MSSM). In Sect. 3 we introduce the complex parameters of the $SU(2)\times U(1)$ invariant potential terms and discuss the diagonalization of the THDM potential which acquires explicit CP-violation. In Sects. 4 and 5 we calculate the Higgs–gauge boson, Higgs–fermion and Higgs self-couplings in the MSSM with CP-violation.

2 Diagonalization of the mass matrix in the general two-Higgs-doublet model

Two representations have been used for the two-doublet Higgs potential. The first representation [2,4],

$$\begin{aligned}V(\varphi_1,\varphi_2) &= \lambda_1\left(\varphi_1^+\varphi_1-\frac{v_1^2}{2}\right)^2+\lambda_2\left(\varphi_2^+\varphi_2-\frac{v_2^2}{2}\right)^2\\ &+\lambda_3\left[\left(\varphi_1^+\varphi_1-\frac{v_1^2}{2}\right)+\left(\varphi_2^+\varphi_2-\frac{v_2^2}{2}\right)\right]^2\\ &+\lambda_4[(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2)-(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)]\\ &+\lambda_5\left[\mathrm{Re}(\varphi_1^+\varphi_2)-\frac{v_1v_2}{2}\mathrm{Re}(\mathrm{e}^{\mathrm{i}\xi})\right]^2\\ &+\lambda_6\left[\mathrm{Im}(\varphi_1^+\varphi_2)-\frac{v_1v_2}{2}\mathrm{Im}(\mathrm{e}^{\mathrm{i}\xi})\right]^2,\end{aligned} \quad (1)$$

where the λ_i are real constants, and the $SU(2)$ doublets $\varphi_{1,2}$ have the components

$$\begin{aligned}\varphi_1 &= \left\{-\mathrm{i}w_1^+,\frac{1}{\sqrt{2}}(v_1+h_1+\mathrm{i}z_1)\right\},\\ \varphi_2 &= \left\{-\mathrm{i}w_2^+,\frac{1}{\sqrt{2}}(v_2+h_2+\mathrm{i}z_2)\right\}.\end{aligned} \quad (2)$$

$w_{1,2}$ are complex fields and $z_{1,2}$, $h_{1,2}$ are real scalar fields. At positive $\lambda_1,\cdots,\lambda_6$ each term of the potential $V(\varphi_1,\varphi_2)$ is obviously positive and its zero minimum is achieved if the vacuum expectation values of $\langle\varphi_1\rangle$, $\langle\varphi_2\rangle$ are taken in the form

$$\langle\varphi_1\rangle=\frac{1}{\sqrt{2}}\{0,v_1\},\quad \langle\varphi_2\rangle=\frac{1}{\sqrt{2}}\{0,v_2\mathrm{e}^{\mathrm{i}\xi}\}. \quad (3)$$

In the case of $\lambda_5=\lambda_6$ (corresponding to the CP-conserving MSSM-like potential, see below) the last two terms in (1) form the modulo squared, and the phase ξ can be removed from the potential by a $U(1)$ rotation of φ_2, which does not affect minimization. In this section we will consider the case $\lambda_5\neq\lambda_6$, $\xi=0$. Substitution of (2) into (1) gives a bilinear form of the mass term with mixed components w_i,h_i,z_i, which can be diagonalized by an orthogonal transformation of the fields in order to define the tree-level masses of the Higgs bosons. In the CP-conserving case the potential terms involving the z_1,z_2 fields from the imaginary parts of the φ_1, φ_2 doublets and the h_1,h_2 fields from their real parts do not mix, so the mass terms are diagonalized by separate two-dimensional rotations of the z_1,z_2 and the h_1,h_2 fields. The resulting spectrum of scalars consists of two charged $H^\pm$, three neutral h, H, A^0 scalar fields, and three Goldstone bosons G. This procedure is described in many papers (for instance, [4,9]). The $w_{1,2}$ sector is diagonalized by the rotation of $w_1,w_2\to H,G$:

$$w_1^\pm=-H^\pm s_\beta+G^\pm c_\beta,\quad w_2^\pm=H^\pm c_\beta+G^\pm s_\beta, \quad (4)$$

defined by the angle

$$\mathrm{tg}\beta=\frac{v_2}{v_1} \quad (5)$$

and leading to the massless G field and the field of the massive charged Higgs boson $H^\pm$, $m_{H\pm}^2=\lambda_4(v_1^2+v_2^2)/2$. The $z_{1,2}$ sector is diagonalized by the rotation $z_1,z_2\to A^0,G'$ defined by the angle β and giving again one massless field G' and the field of a CP-odd Higgs boson A^0 with the mass $m_A^2=\lambda_6(v_1^2+v_2^2)/2$. Finally, the h_1,h_2 sector is diagonalized by the rotation $h_1,h_2\to h,H$ defined by the angle α:

$$\begin{aligned}\sin 2\alpha &= \frac{2m_{12}}{\sqrt{(m_{11}-m_{22})^2+4m_{12}^2}},\\ \cos 2\alpha &= \frac{m_{11}-m_{22}}{\sqrt{(m_{11}-m_{22})^2+4m_{12}^2}},\end{aligned} \quad (6)$$

where

$$\begin{aligned}m_{11} &= \frac{1}{4}[4v_1^2(\lambda_1+\lambda_3)+v_2^2\lambda_5],\\ m_{22} &= \frac{1}{4}[4v_2^2(\lambda_2+\lambda_3)+v_1^2\lambda_5],\\ m_{12} &= \frac{1}{4}(4\lambda_3+\lambda_5)v_1v_2,\end{aligned}$$

giving two massive fields of CP-even Higgs bosons H,h with masses

$$m_{H,h}^2=m_{11}+m_{22}\pm\sqrt{(m_{11}-m_{22})^2+4m_{12}^2}. \quad (7)$$

The diagonal mass term of the scalar fields and their triple and quartic self-interaction vertices can be explicitly obtained by the substitution of the following expressions for λ_i into the potential $V(\varphi_1,\varphi_2)$ (1):

$$\begin{aligned}\lambda_1 &= \frac{1}{2v^2}\frac{1}{c_\beta^2}\left[\frac{s_\alpha}{s_\beta}c_{\alpha-\beta}m_h^2-\frac{c_\alpha}{s_\beta}s_{\alpha-\beta}m_H^2\right]+\frac{c_{2\beta}}{4c_\beta^2}\lambda_5,\\ \lambda_2 &= \frac{1}{2v^2}\frac{1}{s_\beta^2}\left[\frac{c_\alpha}{c_\beta}c_{\alpha-\beta}m_h^2+\frac{s_\alpha}{c_\beta}s_{\alpha-\beta}m_H^2\right]-\frac{c_{2\beta}}{4s_\beta^2}\lambda_5,\\ \lambda_3 &= \frac{1}{2v^2}\left[-\frac{s_{2\alpha}}{s_{2\beta}}m_h^2+\frac{s_{2\alpha}}{s_{2\beta}}m_H^2\right]-\frac{1}{4}\lambda_5,\\ \lambda_4 &= \frac{2}{v^2}m_{H^\pm}^2,\quad \lambda_6=\frac{2}{v^2}m_{A^0}^2,\end{aligned} \quad (8)$$

where we used the notation $v^2 = v_1^2 + v_2^2$, $s_\alpha = \sin\alpha$, $c_\alpha = \cos\alpha$. Diagonalization of the mass term takes place for arbitrary λ_5, which is a free parameter of the model.

The second representation of the Higgs potential,

$$\begin{aligned}U(\varphi_1,\varphi_2) &= -\mu_1^2(\varphi_1^+\varphi_1) - \mu_2^2(\varphi_2^+\varphi_2) \\ &- \mu_{12}^2(\varphi_1^+\varphi_2 + \varphi_2^+\varphi_1) \\ &+ \bar\lambda_1(\varphi_1^+\varphi_1)^2 + \bar\lambda_2(\varphi_2^+\varphi_2)^2 + \bar\lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) \\ &+ \bar\lambda_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) \\ &+ \frac{\bar\lambda_5}{2}[(\varphi_1^+\varphi_2)(\varphi_1^+\varphi_2) + (\varphi_2^+\varphi_1)(\varphi_2^+\varphi_1)], \qquad (9)\end{aligned}$$

originates from the general SUSY action after the integration over Grassman variables and introduction of the soft SUSY-breaking terms (see [4]). It is easy to check that the potentials (1) and (9) are equivalent if the parameters $\bar\lambda_i$, $\mu_1^2, \mu_2^2, \mu_{12}^2$ and λ_i are related by the formulae

$$\begin{aligned}&\bar\lambda_1 = \lambda_1 + \lambda_3, \quad \bar\lambda_2 = \lambda_2 + \lambda_3, \quad \bar\lambda_3 = 2\lambda_3 + \lambda_4, \qquad (10)\\ &\bar\lambda_4 = -\lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2}, \quad \bar\lambda_5 = \frac{\lambda_5}{2} - \frac{\lambda_6}{2}, \quad \mu_{12}^2 = \lambda_5\frac{v_1 v_2}{2}\end{aligned}$$

and

$$\mu_1^2 = \lambda_1 v_1^2 + \lambda_3 v_1^2 + \lambda_3 v_2^2, \quad \mu_2^2 = \lambda_2 v_2^2 + \lambda_3 v_1^2 + \lambda_3 v_2^2. \quad (11)$$

Unlike the potential (1) where the minimization is obvious, the symbolic structure of (9) does not demonstrate evidently its minimum. The substitution of (2) into (9) gives linear terms in the component fields $z_{1,2}$, $h_{1,2}$ (or the physical fields h, H, A) and unless some additional conditions to remove the linear terms are imposed, we are not in the minimum of the potential. So (11), which set to zero the terms which are linear in the component fields, are the minimization conditions. The diagonalization of $U(\varphi_1,\varphi_2)$ takes place for arbitrary parameter μ_{12}^2.

The inverse transformation (10) has the form

$$\begin{aligned}&\lambda_1 = \bar\lambda_1 - \frac{\bar\lambda_3}{2} - \frac{\bar\lambda_4}{2} - \frac{\bar\lambda_5}{2} + \frac{\lambda_5}{2},\\ &\lambda_2 = \bar\lambda_2 - \frac{\bar\lambda_3}{2} - \frac{\bar\lambda_4}{2} - \frac{\bar\lambda_5}{2} + \frac{\lambda_5}{2},\\ &\lambda_3 = \frac{\bar\lambda_3}{2} + \frac{\bar\lambda_4}{2} + \frac{\bar\lambda_5}{2} - \frac{\lambda_5}{2}, \quad \lambda_4 = -\bar\lambda_4 - \bar\lambda_5 + \lambda_5,\\ &\lambda_6 = -2\,\bar\lambda_5 + \lambda_5 \qquad (12)\end{aligned}$$

so the masses of the CP-even scalars and their mixing angle α (6) and (7) in the case of the potential $U(\varphi_1,\varphi_2)$ can be easily obtained using

$$\begin{aligned}m_{11} + m_{22} &= v_1^2\bar\lambda_1 + v_2^2\bar\lambda_2 + \frac{\mu_{12}^2}{s_{2\beta}},\\ m_{11} - m_{22} &= v_1^2\bar\lambda_1 - v_2^2\bar\lambda_2 - \mathrm{ctg}\,2\beta\mu_{12}^2,\\ 2m_{12} &= v_1 v_2(\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5) - \mu_{12}^2. \qquad (13)\end{aligned}$$

The diagonal form of $U(\varphi_1,\varphi_2)$ and the physical scalar boson self-interaction vertices are obtained by substitution of the following expressions for $\bar\lambda_i$ and μ_i into (9):

$$\bar\lambda_1 = \frac{1}{2v^2}\left[\left(\frac{s_\alpha}{c_\beta}\right)^2 m_h^2 + \left(\frac{c_\alpha}{c_\beta}\right)^2 m_H^2 - \frac{s_\beta}{c_\beta^3}\mu_{12}^2\right], \qquad (14)$$

$$\bar\lambda_2 = \frac{1}{2v^2}\left[\left(\frac{c_\alpha}{s_\beta}\right)^2 m_h^2 + \left(\frac{s_\alpha}{s_\beta}\right)^2 m_H^2 - \frac{c_\beta}{s_\beta^3}\mu_{12}^2\right], \qquad (15)$$

$$\bar\lambda_3 = \frac{1}{v^2}\left[2m_{H^\pm}^2 - \frac{\mu_{12}^2}{s_\beta c_\beta} + \frac{s_{2\alpha}}{s_{2\beta}}(m_H^2 - m_h^2)\right], \qquad (16)$$

$$\bar\lambda_4 = \frac{1}{v^2}\left(\frac{\mu_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H^\pm}^2\right), \qquad (17)$$

$$\bar\lambda_5 = \frac{1}{v^2}\left(\frac{\mu_{12}^2}{s_\beta c_\beta} - m_A^2\right), \qquad (18)$$

$$\mu_1^2 = \bar\lambda_1 v_1^2 + (\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5)\frac{v_2^2}{2} - \mu_{12}^2\mathrm{tg}\beta, \qquad (19)$$

$$\mu_2^2 = \bar\lambda_2 v_2^2 + (\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5)\frac{v_1^2}{2} - \mu_{12}^2\mathrm{ctg}\beta. \qquad (20)$$

The conditions (14)–(18) ensure the diagonal form of the mass term expressed in the physical fields $h, H, A, H^\pm$ and (19) and (20) are the minimization conditions. Two parameterizations for the Higgs boson self-interaction vertices can be used in THDM. In the first parameterization [10] μ_{12}^2 is a free parameter and $\bar\lambda_5$ is defined by (18). In the second one $\bar\lambda_5$ is a free parameter and μ_{12}^2 is equal to $s_\beta c_\beta(v^2\bar\lambda_5 + m_A^2)$. Complete sets of Feynman rules (unitary gauge) for the triple (μ_{12}^2 and λ_5 parameterizations) and quartic (μ_{12}^2 parameterization) Higgs boson interactions in the general two-Higgs-doublet model with CP-conservation are shown in Tables 1 and 2[1]. In the case of the MSSM potential at the scale M_{SUSY} (see (29)) $\bar\lambda_5 = 0$ and it follows from (8), (10) and (11) that μ_{12}^2 is fixed and equal to $m_A^2 s_\beta c_\beta$.

Two additional terms of dimension four can be constructed using the complete set of $SU(2)\times U(1)$ invariants $\varphi_1^+\varphi_1$, $\varphi_2^+\varphi_2$, $\mathrm{Re}\varphi_1^+\varphi_2$ and $\mathrm{Im}\varphi_1^+\varphi_2$ (a detailed discussion of all possible potential forms can be found in [12]). These terms are usually added to the $U(\varphi_1,\varphi_2)$ with the parameters $\bar\lambda_6$ and $\bar\lambda_7$

$$\begin{aligned}\bar U(\varphi_1,\varphi_2) &= U(\varphi_1,\varphi_2) \qquad (21)\\ &+ \bar\lambda_6(\varphi_1^+\varphi_1)[(\varphi_1^+\varphi_2) + (\varphi_2^+\varphi_1)]\\ &+ \bar\lambda_7(\varphi_2^+\varphi_2)[(\varphi_1^+\varphi_2) + (\varphi_2^+\varphi_1)].\end{aligned}$$

The diagonal form of $\bar U(\varphi_1,\varphi_2)$ at the local minimum takes place at arbitrary μ_{12}^2, $\bar\lambda_6$, $\bar\lambda_7$ and can be achieved by means of the substitution (14)–(20) with additional $\bar\lambda_6$-, $\bar\lambda_7$-terms on the right-hand side:

$$\begin{aligned}\bar\lambda_1 &= \frac{1}{2v^2}\left[\left(\frac{s_\alpha}{c_\beta}\right)^2 m_h^2 + \left(\frac{c_\alpha}{c_\beta}\right)^2 m_H^2 - \frac{s_\beta}{c_\beta^3}\mu_{12}^2\right]\\ &+ \frac{1}{4}(\bar\lambda_7\mathrm{tg}^3\beta - 3\bar\lambda_6\mathrm{tg}\beta), \qquad (22)\end{aligned}$$

[1] These sets were obtained by means of the LanHEP package [11], see http://theory.sinp.msu.ru/~semenov/lanhep.html Six misprints in the sign of the second term in the factor $(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)$ that occur in [10] are corrected in Table 3 in the expressions for the vertices $hhhh$, $AAAA$, $hhAA$, $H^+H^-H^+H^-$, H^+H^-AA, H^+H^-hh

Table 1. Triple Higgs boson interaction vertices in the general two-Higgs-doublet model, μ_{12} parameterization

Fields in the vertex	Variational derivative of Lagrangian by fields
$h\ h\ h$	$\frac{3e}{M_W s_W s_{2\beta}^2}[-s_{2\beta}(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)m_h^2 + 2c_{\alpha-\beta}^2 c_{\alpha+\beta}\mu_{12}^2]$
$H\ H\ H$	$\frac{3e}{M_W s_W s_{2\beta}^2}[-s_{2\beta}(c_\alpha^3 s_\beta + s_\alpha^3 c_\beta)m_H^2 + 2s_{\alpha-\beta}^2 s_{\alpha+\beta}\mu_{12}^2]$
$H\ H\ h$	$\frac{e s_{\alpha-\beta}}{2M_W s_W s_{2\beta}^2}[-(2m_H^2 + m_h^2)s_{2\alpha}s_{2\beta} + 4(3s_\alpha c_\alpha + s_\beta c_\beta)\mu_{12}^2]$
$H\ h\ h$	$-\frac{e c_{\alpha-\beta}}{2M_W s_W s_{2\beta}^2}[(m_H^2 + 2m_h^2)s_{2\alpha}s_{2\beta} - 4(3s_\alpha c_\alpha - s_\beta c_\beta)\mu_{12}^2]$
$H\ A\ A$	$-\frac{e}{M_W s_W s_{2\beta}^2}[s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + s_{2\beta}^2 c_{\alpha-\beta}m_A^2 - 2s_{\alpha+\beta}\mu_{12}^2]$
$h\ A\ A$	$\frac{e}{M_W s_W s_{2\beta}^2}[s_{2\beta}(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + s_{2\beta}^2 s_{\alpha-\beta}m_A^2 + 2c_{\alpha+\beta}\mu_{12}^2]$
$h\ H^+\ H^-$	$\frac{e}{M_W s_W s_{2\beta}^2}[s_{2\beta}(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + s_{2\beta}^2 s_{\alpha-\beta}m_{H^\pm}^2 + 2c_{\alpha+\beta}\mu_{12}^2]$
$H\ H^+\ H^-$	$-\frac{e}{M_W s_W s_{2\beta}^2}[s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + s_{2\beta}^2 c_{\alpha-\beta}m_{H^\pm}^2 - 2s_{\alpha+\beta}\mu_{12}^2]$

Table 2. Triple Higgs boson interaction vertices in the general two-Higgs-doublet model, λ_5 parameterization

Fields in the vertex	Variational derivative of Lagrangian by fields
$h\ h\ h$	$\frac{3e}{M_W s_W s_{2\beta}}[-(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)m_h^2 + c_{\alpha-\beta}^2 c_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$
$H\ H\ H$	$\frac{3e}{M_W s_W s_{2\beta}}[-(c_\alpha^3 s_\beta + s_\alpha^3 c_\beta)m_H^2 + s_{\alpha-\beta}^2 s_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$
$H\ H\ h$	$\frac{e s_{\alpha-\beta}}{2M_W s_W s_{2\beta}}[-(2m_H^2 + m_h^2)s_{2\alpha} + 2(3s_\alpha c_\alpha + s_\beta c_\beta)(m_A^2 + v^2\lambda_5)]$
$H\ h\ h$	$-\frac{e c_{\alpha-\beta}}{2M_W s_W s_{2\beta}}[(m_H^2 + 2m_h^2)s_{2\alpha} - 2(3s_\alpha c_\alpha - s_\beta c_\beta)(m_A^2 + v^2\lambda_5)]$
$H\ A\ A$	$-\frac{e}{M_W s_W s_{2\beta}}[(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + c_{2\beta}s_{\alpha-\beta}m_A^2 - s_{\alpha+\beta}v^2\lambda_5]$
$h\ A\ A$	$\frac{e}{M_W s_W s_{2\beta}}[(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + c_{2\beta}c_{\alpha-\beta}m_A^2 + c_{\alpha+\beta}v^2\lambda_5]$
$h\ H^+\ H^-$	$\frac{e}{M_W s_W s_{2\beta}}[(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)m_h^2 + s_{\alpha-\beta}m_{H^\pm}^2 + c_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$
$H\ H^+\ H^-$	$-\frac{e}{M_W s_W s_{2\beta}}[(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + c_{\alpha-\beta}m_{H^\pm}^2 - s_{\alpha+\beta}(m_A^2 + v^2\lambda_5)]$

$$\bar\lambda_2 = \frac{1}{2v^2}\left[\left(\frac{c_\alpha}{s_\beta}\right)^2 m_h^2 + \left(\frac{s_\alpha}{s_\beta}\right)^2 m_H^2 - \frac{c_\beta}{s_\beta^3}\mu_{12}^2\right] + \frac{1}{4}(\bar\lambda_6 \mathrm{ctg}^3\beta - 3\bar\lambda_7 \mathrm{ctg}\beta), \quad (23)$$

$$\bar\lambda_3 = \frac{1}{v^2}\left[2m_{H^\pm}^2 - \frac{\mu_{12}^2}{s_\beta c_\beta} + \frac{s_{2\alpha}}{s_{2\beta}}(m_H^2 - m_h^2)\right] - \frac{\bar\lambda_6}{2}\mathrm{ctg}\beta - \frac{\bar\lambda_7}{2}\mathrm{tg}\beta, \quad (24)$$

$$\bar\lambda_4 = \frac{1}{v^2}\left(\frac{\mu_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H^\pm}^2\right) - \frac{\bar\lambda_6}{2}\mathrm{ctg}\beta - \frac{\bar\lambda_7}{2}\mathrm{tg}\beta, \quad (25)$$

$$\bar\lambda_5 = \frac{1}{v^2}\left(\frac{\mu_{12}^2}{s_\beta c_\beta} - m_A^2\right) - \frac{\bar\lambda_6}{2}\mathrm{ctg}\beta - \frac{\bar\lambda_7}{2}\mathrm{tg}\beta, \quad (26)$$

$$\mu_1^2 = \bar\lambda_1 v_1^2 + (\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5)\frac{v_2^2}{2} - \mu_{12}^2 \mathrm{tg}\beta + \frac{v^2 s_\beta^2}{2}(3\bar\lambda_6 \mathrm{ctg}\beta + \bar\lambda_7 \mathrm{tg}\beta), \quad (27)$$

$$\mu_2^2 = \bar\lambda_2 v_2^2 + (\bar\lambda_3 + \bar\lambda_4 + \bar\lambda_5)\frac{v_1^2}{2} - \mu_{12}^2 \mathrm{ctg}\beta + \frac{v^2 c_\beta^2}{2}(\bar\lambda_6 \mathrm{ctg}\beta + 3\bar\lambda_7 \mathrm{tg}\beta). \quad (28)$$

Our expressions for the redefined $\bar\lambda_4$ and $\bar\lambda_5$ are the same as given in [13].

The potentials (1) and (9) can be reduced to the MSSM potential in some regions of the parameter space which we are going to discuss. The potential $V(\varphi_1, \varphi_2)$ (1) has eight parameters: two VEV's, v_1, v_2, and six couplings, λ_i $(i = 1, \cdots, 6)$. Eight parameters of the potential $U(\varphi_1, \varphi_2)$ in (9), μ_1, μ_2, μ_{12} and $\bar\lambda_i$ $(i = 1, \cdots, 5)$, can be found using (10) and (11). From the other side, in the Higgs sector we have eight physical parameters: the mixing angle β and W-boson mass m_W, the mixing angle α, the parameter μ_{12}^2 and four masses of the scalars m_h, m_H, m_A, $m_{H^\pm}$. The m_W is fixed experimentally maintaining the constraint on the v_1, v_2, $v^2 = v_1^2 + v_2^2 = 4m_W^2/e^2 \cdot \sin^2\theta_W$ which follows from the Higgs kinetic term $D_\mu\varphi D^\mu\varphi$ ($g = e/\sin\theta_W$; θ_W is the Weinberg angle). So the Higgs sector of THDM with the potentials (1) or (9) is described by a seven-dimensional parameter space. In the case of the superpotential five additional constraints are imposed, relating all Higgs boson self-couplings $\bar\lambda_i$, $(i = 1, \cdots, 5)$ to the gauge coupling constants at the energy scale M_{SUSY} [14]:

$$\bar\lambda_1^{\mathrm{SUSY}} = \bar\lambda_2^{\mathrm{SUSY}} = \frac{g^2 + g_1^2}{8}, \quad \bar\lambda_3^{\mathrm{SUSY}} = \frac{g^2 - g_1^2}{4}, \quad \bar\lambda_4^{\mathrm{SUSY}} = -\frac{g^2}{2}, \quad \bar\lambda_5^{\mathrm{SUSY}} = 0. \quad (29)$$

Table 3. Quartic Higgs boson interaction vertices in the general two-Higgs-doublet model, μ_{12} parameterization

Fields in the vertex	Variational derivative of Lagrangian by fields
$h \quad h \quad h \quad h$	$-\frac{3}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[4s_{2\beta}(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)^2 m_h^2 + s_{2\beta}s_{2\alpha}^2 c_{\alpha-\beta}^2 m_H^2 - 8c_{\alpha-\beta}^2 c_{\alpha+\beta}^2 \mu_{12}^2]$
$H \quad H \quad H \quad H$	$\frac{3}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-4s_{2\beta}(c_\alpha^3 s_\beta + s_\alpha^3 c_\beta)^2 m_H^2 + s_{2\beta}s_{2\alpha}^2 s_{\alpha-\beta}^2 m_h^2 + 8s_{\alpha-\beta}^2 s_{\alpha+\beta}^2 \mu_{12}^2]$
$A^0 \quad A^0 \quad A^0 \quad A^0$	$-3\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)^2 m_H^2 s_{2\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)^2 m_h^2 - 2c_{2\beta}^2 \mu_{12}^2]$
$H \quad H \quad H \quad h$	$-\frac{3}{4}\frac{e^2 s_{2\alpha}s_{\alpha-\beta}}{M_W^2 s_w^2 s_{2\beta}^3}[2s_{2\beta}(c_\alpha^3 s_\beta + s_\alpha^3 c_\beta)m_H^2 + s_{2\beta}s_{2\alpha}c_{\alpha-\beta}m_h^2 - 4s_{\alpha+\beta}]\mu_{12}^2$
$H \quad h \quad h \quad h$	$-\frac{3}{4}\frac{e^2 s_{2\alpha}c_{\alpha-\beta}}{M_W^2 s_w^2 s_{2\beta}^3}[2s_{2\beta}(c_\alpha^3 c_\beta - s_\alpha^3 s_\beta)m_h^2 + s_{2\beta}s_{2\alpha}s_{\alpha-\beta}m_H^2 - 4c_{\alpha+\beta}]\mu_{12}^2$
$H \quad H \quad h \quad h$	$-\frac{1}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-s_{2\beta}s_{2\alpha}(3s_{2\alpha}s_{\alpha-\beta}^2 - 4s_{\alpha-\beta}c_{\alpha+\beta} - 2s_{\alpha+\beta}s_{\alpha-\beta}c_{\alpha-\beta})m_h^2$
	$+s_{2\beta}s_{2\alpha}(s_{2\beta} + 3s_{2\alpha}s_{\alpha-\beta}^2)m_H^2 - 8(3s_\alpha^2 c_\alpha^2 - s_\beta^2 c_\beta^2)\mu_{12}^2]$
$H \quad H \quad A^0 \quad A^0$	$\frac{1}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-2s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)m_h^2 - 2s_{2\beta}^3 c_{\alpha-\beta}^2 m_A^2$
	$-s_{2\beta}(s_{2\alpha}s_{2\beta} + 3s_{\alpha-\beta}^2 s_{\alpha+\beta}^2 - s_{2\beta}s_{\alpha-\beta}^2)m_H^2 + 4(c_{2\beta}^2 s_{\alpha-\beta}^2 + s_{\alpha+\beta}^2)\mu_{12}^2]$
$h \quad h \quad A^0 \quad A^0$	$\frac{1}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-s_{2\beta}^2(4c_{2\beta}c_{2\alpha} + 3s_{\alpha-\beta}^2 s_{\alpha+\beta}^2 + s_{\alpha-\beta}^4)m_h^2 - 2s_{2\beta}^3 s_{\alpha-\beta}^2 m_A^2$
	$-2s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + 2(s_{2\beta}^2 s_{\alpha-\beta}^2 + 4(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)^2)\mu_{12}^2]$
$H \quad A^0 \quad A^0 \quad h$	$\frac{1}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-2s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)m_h^2 + s_{2\beta}^3 s_{\alpha-\beta}c_{\alpha-\beta}m_A^2$
	$-2s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + 2(2s_{2\alpha}c_{2\beta} - s_{2\beta}s_{\alpha-\beta}c_{\alpha-\beta})\mu_{12}^2]$
$H^+ \quad H^+ \quad H^- \quad H^-$	$-2\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[s_{2\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)m_h^2 + s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 - 2c_{2\beta}^2 \mu_{12}^2]$
$H^+ \quad H^- \quad A^0 \quad A^0$	$-\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[s_{2\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)m_h^2 + s_{2\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 - 2c_{2\beta}^2 \mu_{12}^2]$
$H^+ \quad H^- \quad h \quad h$	$\frac{1}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-s_{2\beta}(4c_{2\alpha}c_{2\beta} + 3s_{\alpha-\beta}^2 s_{\alpha+\beta}^2 + s_{\alpha-\beta}^4)m_h^2 - 2s_{2\beta}^3 s_{\alpha-\beta}^2 m_{H^\pm}^2$
	$-2s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)m_H^2 + 2(s_{2\beta}^2 s_{\alpha-\beta}^2 + 4(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)^2 \mu_{12}^2]$
$H^+ \quad H^- \quad H \quad H$	$\frac{1}{4}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-2s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)m_h^2 - 2s_{2\beta}^3 c_{\alpha-\beta}^2 m_{H^\pm}^2$
	$+s_{2\beta}(s_{2\alpha}s_{2\beta} - 3s_{\alpha-\beta}^2 s_{\alpha+\beta}^2 + s_{\alpha-\beta}^4)m_H^2 + 4(c_{2\beta}^2 s_{\alpha-\beta}^2 + s_{\alpha+\beta}^2)\mu_{12}^2]$
$H \quad H^+ \quad H^- \quad h$	$\frac{1}{2}\frac{e^2}{M_W^2 s_w^2 s_{2\beta}^3}[-s_{2\beta}s_{2\alpha}c_{\alpha-\beta}(c_\alpha c_\beta^3 - s_\alpha s_\beta^3)m_h^2 + s_{2\beta}^3 s_{\alpha-\beta}c_{\alpha-\beta}m_{H^\pm}^2$
	$-s_{2\beta}s_{2\alpha}s_{\alpha-\beta}(c_\alpha s_\beta^3 + s_\alpha c_\beta^3)m_H^2 + 2(2s_{2\alpha}c_{2\beta} - s_{2\beta}^2 s_{\alpha-\beta}c_{\alpha-\beta})\mu_{12}^2]$

It follows that the four Higgs boson masses and the two mixing angles are defined by two independent parameters. One can choose, for example, the r_1, r_2 parameterization [15] ($r_{1,2} = m_{h,H}^2/m_Z^2$) or the well-known m_A, tgβ parameterization. In order to reduce the general two-Higgs-doublet model vertices to MSSM at the scale $M_{\rm SUSY}$ it is convenient to use the α, β parameterization:

$$m_h^2 = m_Z^2 c_{2\beta}\frac{s_{\alpha+\beta}}{s_{\alpha-\beta}}, \quad m_H^2 = m_Z^2 c_{2\beta}\frac{c_{\alpha+\beta}}{c_{\alpha-\beta}},$$
$$m_A^2 = m_Z^2 \frac{s_{2(\alpha+\beta)}}{s_{2(\alpha-\beta)}}, \quad \mu_{12}^2 = m_A^2 s_\beta c_\beta. \tag{30}$$

Substitution of these expressions to the vertex factors in Tables 1 and 2 after trivial trigoniometric transformations reduces them [10] to simpler MSSM factors [4]. However, (30) are no longer valid at the energy scale m_W where the $\bar\lambda_i^{\rm SUSY}$ couplings and the masses of the Higgs bosons are significantly changed by radiative corrections and the effective two-Higgs-doublet potential should be described in the complete seven-dimensional parameter space. Practical calculations of the radiatively corrected masses and/or couplings can be conveniently carried out using the results of the two approaches, the renormalization group (the HMSUSY package [16] or the analytical representation [17]), and diagrammatic (the FeynHiggsFast package [18]). Two different parameterizations can be used for these approaches.

In the RG approach it seems convenient to use the two-Higgs-doublet model parameter space m_A, tgβ, $\bar\lambda_1, \cdots, \bar\lambda_5$. In the following we shall take into account the $\bar\lambda_6$- and $\bar\lambda_7$-terms defined by (21), so the parameter space will be nine-dimensional. The RG evolution of the coupling constants $\bar\lambda_i$ from the energy scale $M_{\rm SUSY}$ to the electroweak scale m_W defines the $\bar\lambda_1, \cdots, \bar\lambda_5$ in (22)–(26) and the parameters $\bar\lambda_6$, $\bar\lambda_7$. At a given m_A, tgβ, $\bar\lambda_6$, $\bar\lambda_7$ the parameters μ_{12}^2 and $m_{H^\pm}$ are fixed by the conditions (25) and (26), the parameters μ_1^2 and μ_2^2 are fixed by (27) and (28), and α, m_h and m_H can be found using (22)–(24). If we denote the deviation from the coupling $\bar\lambda_i^{\rm SUSY}$ at the MSSM scale by $\Delta\bar\lambda_i$,

$$2(\bar\lambda_{1,2}^{\rm SUSY} - \bar\lambda_{1,2}) = \Delta\bar\lambda_{1,2}, \quad \bar\lambda_{3,4}^{\rm SUSY} - \bar\lambda_{3,4} = \Delta\bar\lambda_{3,4},$$
$$-\bar\lambda_{5,6,7} = \Delta\bar\lambda_{5,6,7},$$

we find the mixing angle (introducing the notation $g_1^2 + g^2 = g^2 m_Z^2/m_W^2$, $g^2 - g_1^2 = g^2(2 - m_Z^2/m_W^2)$ while using (22)–(26))

$$\text{tg}2\alpha = \tag{31}$$

$$\frac{s_{2\beta}(m_A^2+m_Z^2)+v^2((\Delta\bar\lambda_3+\Delta\bar\lambda_4)s_{2\beta}+2c_\beta^2\Delta\bar\lambda_6+2s_\beta^2\Delta\bar\lambda_7)}{c_{2\beta}(m_A^2-m_Z^2)+v^2(\Delta\bar\lambda_1 c_\beta^2-\Delta\bar\lambda_2 s_\beta^2-\Delta\bar\lambda_5 c_{2\beta}+(\Delta\bar\lambda_6-\Delta\bar\lambda_7)s_{2\beta})};$$

CP-even Higgs boson masses and the μ_{12}^2 parameter

$$\begin{aligned}
m_H^2 &= c_{\alpha+\beta}^2 m_Z^2 + s_{\alpha-\beta}^2 m_A^2 \\
&- v^2\Big(\Delta\bar\lambda_1 c_\alpha^2 c_\beta^2 + \Delta\bar\lambda_2 s_\alpha^2 s_\beta^2 \\
&+ 2(\Delta\bar\lambda_3+\Delta\bar\lambda_4)c_\alpha c_\beta s_\alpha s_\beta \\
&+ \Delta\bar\lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2)\Big) \\
&+ 2s_{\alpha+\beta}(\Delta\bar\lambda_6 c_\alpha c_\beta + \Delta\bar\lambda_7 s_\alpha s_\beta), \\
m_h^2 &= s_{\alpha+\beta}^2 m_Z^2 + c_{\alpha-\beta}^2 m_A^2 \\
&- v^2\Big(\Delta\bar\lambda_1 s_\alpha^2 c_\beta^2 + \Delta\bar\lambda_2 c_\alpha^2 s_\beta^2 \\
&- 2(\Delta\bar\lambda_3+\Delta\bar\lambda_4)c_\alpha c_\beta s_\alpha s_\beta \\
&+ \Delta\bar\lambda_5(s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2)\Big) \\
&- 2c_{\alpha+\beta}(\Delta\bar\lambda_6 s_\alpha c_\beta - \Delta\bar\lambda_7 c_\alpha s_\beta),
\end{aligned} \tag{32}$$

$$m_{H^\pm}^2 = m_W^2 + m_A^2 - \frac{v^2}{2}(\Delta\bar\lambda_5 - \Delta\bar\lambda_4), \tag{33}$$

$$\mu_{12}^2 = s_\beta c_\beta\left[m_A^2 - \frac{v^2}{2}(2\Delta\bar\lambda_5 + \Delta\bar\lambda_6 \mathrm{ctg}\beta + \Delta\bar\lambda_7 \mathrm{tg}\beta)\right],$$

with the minimization conditions

$$\begin{aligned}
\mu_1^2 &= \frac{1}{2}m_Z^2 c_{2\beta} - \mu_{12}^2 \mathrm{tg}\beta \\
&- \frac{v^2}{2}\Big[\Delta\bar\lambda_1 c_\beta^2 + (\Delta\bar\lambda_3+\Delta\bar\lambda_4+\Delta\bar\lambda_5)s_\beta^2 + 3\Delta\bar\lambda_6 s_\beta c_\beta \\
&+ \Delta\bar\lambda_7\frac{s_\beta^3}{c_\beta}\Big], \\
\mu_2^2 &= -\frac{1}{2}m_Z^2 c_{2\beta} - \mu_{12}^2 \mathrm{ctg}\beta \\
&- \frac{v^2}{2}\Big[\Delta\bar\lambda_2 s_\beta^2 + (\Delta\bar\lambda_3+\Delta\bar\lambda_4+\Delta\bar\lambda_5)c_\beta^2 + \Delta\bar\lambda_6\frac{c_\beta^3}{s_\beta} \\
&+ 3\Delta\bar\lambda_7 s_\beta c_\beta\Big].
\end{aligned} \tag{34}$$

These expressions can be straightforwardly used to calculate the radiatively corrected masses of Higgs bosons and the mixing angle α in the MSSM with the help of a solution of the RG equations for $\bar\lambda_1, \cdots, \bar\lambda_7$. Apparently, in the RG approach Feynman rules in terms of $\bar\lambda_i$ couplings are more convenient than rules in terms of Higgs particle masses.

In the diagrammatic approaches to the calculation of the radiatively corrected masses [18] the corrections to m_h, m_H, m_A and $m_{H^\pm}$ are extracted from the renormalized Higgs boson self-energies (usually radiative corrections to only the CP-even Higgs boson masses are calculated). The set of $7+2$ independent parameters inherent for the diagrammatic approaches could be m_A, tgβ, α, μ_{12}^2, m_h, m_H, $m_{H^\pm}$, and $\bar\lambda_6, \bar\lambda_7$. At a given m_A, tgβ, $\bar\lambda_6, \bar\lambda_7$ the μ_{12}^2 parameter can be fixed at the value $m_A^2 s_\beta c_\beta$, and α can be calculated using the renormalized self-energies correction [18] to the relation valid at the M_{SUSY} scale $m_A^2 + m_Z^2 = -s_{2\alpha}/s_{2\beta}(m_H^2 - m_h^2)$. Then $\bar\lambda_4$ is defined by (25) and $\bar\lambda_1, \cdots, \bar\lambda_3$ can be found using (22)–(24). In the diagrammatic calculations Feynman rules in terms of the radiatively corrected Higgs boson masses look more natural. A substitution of the radiatively corrected Higgs masses to the tree-level Higgs vertex factors is expected to give results very close to those obtained from the loop corrections to the Higgs vertex at the SUSY scale (see the discussion in the last of [8]). It has been shown in [19] by the example of *hhh* and *hhhh* vertices (and for the case of a diagonal third generation squark mass matrix) that large radiative corrections to the vertex factors calculated diagrammatically can be absorbed in the radiatively corrected Higgs boson masses.

Other parameterizations in the two-Higgs-doublet model are of course possible, but they should be carefully introduced to respect the minimization and diagonalization conditions (22)–(28). The introduction of scalar particle masses and mixing angles inconsistent with them violates either the diagonalizaton of the potential or its $SU(2)$ invariance, even if the minimization conditions remain valid.

3 CP-violation in the two-Higgs-doublet model

CP-transformation of the scalar doublet $CP\varphi P^+C^+ = \zeta_{CP}\overset{*}{\varphi}{}^+$ (the phase factor $|\zeta_{CP}|=1$) changes the sign of the imaginary part Im$(\varphi_1^+\varphi_2)$ in the λ_6-term of the potential in (1), so if $\lambda_5 \neq \lambda_6$, $\lambda_6 \neq 0$ and $\xi \neq 0$, CP-symmetry is broken there explicitly. In other words, the dimension two terms of (1) appear with the complex parameter μ_{12}^2:

$$\begin{aligned}
&\frac{\lambda_5}{4}[\varphi_1^+\varphi_2 + \varphi_2^+\varphi_1 - v_1v_2\cos\xi]^2 \\
&+ \frac{\lambda_6}{4}[-\mathrm{i}(\varphi_1^+\varphi_2 - \varphi_2^+\varphi_1) - v_1v_2\sin\xi]^2 \\
&\Rightarrow \left(\frac{\lambda_5}{4} - \frac{\lambda_6}{4}\right)[(\varphi_1^+\varphi_2)^2 + (\varphi_2^+\varphi_1)^2] \\
&+ \left(\frac{\lambda_5}{2} + \frac{\lambda_6}{2}\right)\varphi_1^+\varphi_2\varphi_2^+\varphi_1 \\
&- \frac{v_1v_2}{2}(\lambda_5\cos\xi - \mathrm{i}\lambda_6\sin\xi)\varphi_1^+\varphi_2 \\
&- \frac{v_1v_2}{2}(\lambda_5\cos\xi + \mathrm{i}\lambda_6\sin\xi)\varphi_2^+\varphi_1,
\end{aligned} \tag{35}$$

so we find

$$\mu_{12}^2 = \frac{v_1v_2}{2}(\lambda_5\cos\xi - \mathrm{i}\lambda_6\sin\xi). \tag{36}$$

CP is (softly) broken by the $\mu_{12}^2\varphi_1^+\varphi_2 + \overset{*}{\mu}{}_{12}^2\varphi_2^+\varphi_1$-terms. In this special case when the same phase ξ is involved both in the potential and the vacuum expectation value of φ_2, the diagonalization and minimization of the CP-transformed potential become less transparent. It is more convenient to analyze the potential form (21) for the general case of complex parameters with arbitrary phases. In the following we shall consider the hermitian potential which is the generalization of (21):

$$\begin{aligned}\bar U(\varphi_1,\varphi_2) = \frac{1}{2}\Big[&- \mu_1^2(\varphi_1^+\varphi_1) - \overset{*}{\mu_1^2}(\varphi_1^+\varphi_1) - \mu_2^2(\varphi_2^+\varphi_2) \\ &- \overset{*}{\mu_2^2}(\varphi_2^+\varphi_2)\Big] - \mu_{12}^2(\varphi_1^+\varphi_2) - \overset{*}{\mu_{12}^2}(\varphi_2^+\varphi_1) \\ &+ \frac{1}{2}\Big[\bar\lambda_1(\varphi_1^+\varphi_1)^2 + \overset{*}{\bar\lambda_1}(\varphi_1^+\varphi_1)^2 + \bar\lambda_2(\varphi_2^+\varphi_2)^2 \\ &+ \overset{*}{\bar\lambda_2}(\varphi_2^+\varphi_2)^2 \\ &+ \bar\lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \overset{*}{\bar\lambda_3}(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) \\ &+ \bar\lambda_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) + \overset{*}{\bar\lambda_4}(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1)\Big] \\ &+ \frac{\bar\lambda_5}{2}(\varphi_1^+\varphi_2)(\varphi_1^+\varphi_2) + \frac{\overset{*}{\bar\lambda_5}}{2}(\varphi_2^+\varphi_1)(\varphi_2^+\varphi_1) \\ &+ \bar\lambda_6(\varphi_1^+\varphi_1)(\varphi_1^+\varphi_2) + \overset{*}{\bar\lambda_6}(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_1) \\ &+ \bar\lambda_7(\varphi_2^+\varphi_2)(\varphi_1^+\varphi_2) + \overset{*}{\bar\lambda_7}(\varphi_2^+\varphi_2)(\varphi_2^+\varphi_1); \quad (37)\end{aligned}$$

$\overset{*}{a}$ denotes the complex conjugated of a. The potential terms with complex parameters μ_{12}^2 and $\bar\lambda_5, \bar\lambda_6, \bar\lambda_7$ explicitly violate CP-invariance. With the help of $U(1)_Y$ hypercharge symmetry of the model the phases θ_μ and $\theta_{5,6,7}$ of the complex parameters μ_{12}^2 and $\bar\lambda_{5,6,7}$ can be removed by the phase rotation of scalar doublets $\varphi_{1,2}$. In the case when v_1 in (3) is taken real and positive, and η is the overall phase of the φ_2 doublet, the conditions to remove the explicit phases from the potential are [20]

$$\begin{aligned}&\theta_\mu - \eta = n_\mu\pi, \quad \theta_5 - 2\eta = n_5\pi, \\ &\theta_{6,7} - \eta = n_{6,7}\pi, \qquad (38)\end{aligned}$$

where n_μ, n_i ($i = 5,6,7$) are arbitrary integer numbers. Or equivalently, in terms of complex parameters the conditions for the absence of explicit CP-violation in the effective potential (37) are [5]

$$\begin{aligned}&\mathrm{Im}(\mu_{12}^4\,\overset{*}{\bar\lambda_5}) = 0, \quad \mathrm{Im}(\mu_{12}^2\,\overset{*}{\bar\lambda_6}) = 0, \\ &\mathrm{Im}(\mu_{12}^2\,\overset{*}{\bar\lambda_7}) = 0. \qquad (39)\end{aligned}$$

The phase of complex μ_{12}^2 can always be rotated away, so μ_{12}^2 can be taken real. In the scenario of fine-tuning for the phases, when the conditions (38) or (39) are satisfied, the $\bar\lambda_i$ are also real. Otherwise $\bar\lambda_i$ will be redefined after a phase rotation, keeping explicitly CP-violating terms.

If the phases of μ_{12}^2, $\bar\lambda_i$ are rotated away so there are no explicitly CP-non-invariant potential terms, CP-invariance can nevertheless be broken spontaneously. Using the convention that v_1 and v_2 in (3) are real and positive and selecting the ξ phase-dependent terms of (37), which are of the form $a\cos^2\xi + b\cos\xi$, where $a = \bar\lambda_5 v_1^2 v_2^2/2$ and $b = (\bar\lambda_6 v_1^2/2 + \bar\lambda_7 v_2^2/2 - \mu_{12}^2)v_1 v_2$ ($0 \le \xi \le \pi$), one can find their minimum (see the appendix of [20]) at $\bar\lambda_5 \ge 0$ and

$$\cos\xi = \frac{\mu_{12}^2 - \dfrac{v^2}{2}\bar\lambda_6 c_\beta^2 - \dfrac{v^2}{2}\bar\lambda_7 s_\beta^2}{\bar\lambda_5 v^2 s_\beta c_\beta} \qquad (40)$$

(the special case $\mu_{12}^2 = 0$ was found in [1,3]). If $| \mu_{12}^2 - (v^2/2)\bar\lambda_6 c_\beta^2 - (v^2/2)\bar\lambda_7 s_\beta^2 | \ge \bar\lambda_5 v^2 s_\beta c_\beta$, then the spontaneously CP-violating extremum does not exist and the minimum of the potential occurs at the endpoints $\cos\xi = \pm 1$. For $b > 0$ the minimum is at $\xi = \pi$ and for $b < 0$ the minimum is at $\xi = 0$. If $\bar\lambda_5 < 0$, the extremum inside the interval $|\cos\xi| < 1$ is the maximum, so the local minima are at the endpoints $\xi = 0, \pi$. The case $\xi = 0$ is reduced to the case $\xi = \pi$ by the change of sign for $\bar\lambda_6, \bar\lambda_7, \mu_{12}^2$ (change of sign for the b). The special case of CP-conservation occurs at $\xi = \pi/2$ when $\mu_{12}^2 = (v^2/2)\bar\lambda_6 c_\beta^2 + (v^2/2)\bar\lambda_7 s_\beta^2$. In this case, of purely imaginary $\langle\varphi_2\rangle$ [21], our diagonalization procedure must be reconsidered. For instance, CP-even Higgs mass eigenstates are formed in this case by different orthogonal linear combinations of the real and imaginary parts of the scalar doublets, $\mathrm{Re}\varphi_1$ and $\mathrm{Im}\varphi_2$.

The substitution of (26) into the no-extremum condition $|\mu_{12}^2 - (v^2/2)\bar\lambda_6 c_\beta^2 - (v^2/2)\bar\lambda_7 s_\beta^2| \ge \bar\lambda_5 v^2 s_\beta c_\beta$ gives $m_A^2 \ge 0$. In the case $\bar\lambda_5 < 0$ the absolute minimum of the potential occurs at $\xi = 0$ (but not $\xi = \pi$) if $\mu_{12}^2 - (v^2/2)\bar\lambda_6 c_\beta^2 - (v^2/2)\bar\lambda_7 s_\beta^2 \ge 0$, which gives, in combination with (26), $m_A^2 \ge | \bar\lambda_5 | v^2$. Even if $\bar\lambda_5$ is of the order of 1, the latter condition is valid when m_A is in the mass range of the order of or greater than v. In the case of real parameters μ_{12}^2, $\bar\lambda_i$, it is not straightforward to combine spontaneous CP-violation with our procedure of diagonalization. In the case of complex parameters the situation may be changed. The extremum conditions can be found from the study of the fourth power equation with the coefficients depending on the real and imaginary parts of μ_{12}^2, $\bar\lambda_i$. Nevertheless we are not going to consider spontaneous CP-violation further on. With real parameters μ_{12}^2, $\bar\lambda_i$ and in the absence of spontaneous CP-violation the minimum of the potential (37) can be taken at $\xi = 0$. At the same time, insofar as the physical motivation of the fine-tuning conditions (38) and (39) for CP-conservation is not available, in the following we consider the general case of diagonalization and minimization of the two-Higgs-doublet potential with complex parameters.

For the diagonalization of (37) in the ground state we used the ansatz (22)–(28) to be taken for the real parts of the parameters. The real part of the parameter μ_{12}^2 is expressed through the real parts of $\bar\lambda_{5,6,7}$ using (26):

$$\begin{aligned}\mathrm{Re}\mu_{12}^2 = &\, m_A^2 s_\beta c_\beta \qquad (41) \\ &+ v^2\left(s_\beta c_\beta \mathrm{Re}\bar\lambda_5 + \frac{1}{2}c_\beta^2\mathrm{Re}\bar\lambda_6 + \frac{1}{2}s_\beta^2\mathrm{Re}\bar\lambda_7\right),\end{aligned}$$

defining also the real parts of $\bar\lambda_{1,2,3,4}$ and μ_1^2, μ_2^2 by means of (22)–(25), and (27) and (28). The substitution of complex μ_i and $\bar\lambda_i$ into the potential (37) leads to the linear term and the non-diagonal mass term which are dependent on the imaginary parts of $\mu_{12}^2, \bar{\lambda}_i$:

$$\bar{U}(\varphi_1,\varphi_2) = c_0 A + c_1 hA + c_2 HA + \frac{m_h^2}{2}H^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^+H^- + \text{third and fourth order terms in } h, H, A, H^\pm, \tag{42}$$

where

$$\begin{aligned} c_0 &= -v\mathrm{Im}\mu_{12}^2 + \frac{v^3}{2}s_\beta c_\beta \mathrm{Im}\bar{\lambda}_5 + \frac{v^3}{2}(c_\beta^2 \mathrm{Im}\bar{\lambda}_6 + s_\beta^2 \mathrm{Im}\bar{\lambda}_7), \\ c_1 &= -s_{\alpha-\beta}\mathrm{Im}\mu_{12}^2 + \frac{v^2}{4}(s_{2\beta}s_{\alpha-\beta} - 2c_{\alpha+\beta})\mathrm{Im}\bar{\lambda}_5 \\ &\quad - \frac{v^2}{2}(s_\beta c_\beta c_{\alpha-\beta} - 3s_\alpha c_\beta)\mathrm{Im}\bar{\lambda}_6 \\ &\quad + \frac{v^2}{2}(s_\beta c_\beta c_{\alpha-\beta} - 3c_\alpha s_\beta)\mathrm{Im}\bar{\lambda}_7, \\ c_2 &= c_{\alpha-\beta}\mathrm{Im}\mu_{12}^2 + \frac{v^2}{4}(c_{2\beta}s_{\alpha-\beta} - 3s_{\alpha+\beta})\mathrm{Im}\bar{\lambda}_5 \\ &\quad - \frac{v^2}{2}(c_\beta^2 c_{\alpha-\beta} + 2c_\alpha c_\beta)\mathrm{Im}\bar{\lambda}_6 \\ &\quad - \frac{v^2}{2}(s_\beta^2 c_{\alpha-\beta} + 2s_\alpha s_\beta)\mathrm{Im}\bar{\lambda}_7. \end{aligned} \tag{43}$$

In the case of the CP-conservation of (38) and (39), the linear and non-diagonal second order terms hA and HA do not appear because all imaginary parts of the parameters can be removed. The linear term in A demonstrates that after the introduction of complex couplings we may find ourselves to be out of a local minimum of the potential $\bar{U}(\varphi_1,\varphi_2)$.

The minimization condition for the imaginary parts $c_0 = 0$ must be imposed. If $\bar{\lambda}_i$, $i = 5,6,7$ are zero (this occurs in THDM if we additionally introduce a global $U(1)_Q$ symmetry, softly broken by the dimension two μ_{12}^2-term [22]), the imaginary part of μ_{12}^2 can be removed by a phase rotation of φ_2, so the tree-level potential is CP-invariant. In the general THDM with non-zero parameters the phase rotation of φ_2 which removes $\mathrm{Im}\mu_{12}^2$ redefines $\bar{\lambda}_i$, $i = 5,6,7$. These simple observations for the THDM potential isolated from any other physical fields are no longer trivial if, keeping in mind the MSSM, we switch on the interaction of φ_1 and φ_2 with scalar quarks. CP-invariance of the latter is (softly) broken by the dimension three terms with the Higgs mixing parameter μ and the trilinear parameters $A_{t,b}$ of the form $f_{1,2}A_{b,t}\varphi_{1,2}^0 \tilde{q}_R^* \tilde{q}_L$, $\mu f_{2,1}\varphi_{1,2}^0 \tilde{q}_L^* \tilde{q}_R$ ($q = \tilde{t}, \tilde{b}$; $\varphi_{1,2}^0$ are the neutral components of the Higgs doublets, $f_{1,2} = \sqrt{2}m_{b,t}/v_{1,2}$). Then the quartic scalar interaction parameters $\bar{\lambda}_i$, $i = 5,6,7$, are affected by radiative corrections from the one-loop diagrams with scalar quarks of the order of $\mu^2 A^2/M_{\rm SUSY}^4$, $\mu^3 A/M_{\rm SUSY}^4$ and $\mu A^3/M_{\rm SUSY}^4$ [5], so the phases of $\bar{\lambda}_i$, $i = 5,6,7$, are defined by the phases of the complex μ and A, thus constraining the phase of μ_{12}^2 in powers of the conditions (41) and (43). In the case of the Born level MSSM potential with a global $U(1)_Q$ symmetry when $\bar{\lambda}_i = 0$ ($i = 5,6,7$) the complex μ_{12}^2 parameter can still appear beyond the tree-level due to the same CP-violating Yukawa interactions of the scalar quarks with the Higgs fields. This possibility of the μ_{12}^2 phase induced in higher orders by radiative corrections calculated diagrammatically has been considered in [23]. The restoration of the potential minimum can be achieved by means of the opposite sign quantum correction term, originating from the tadpole diagrams with the pseudoscalar A connected to the squark loops[2].

In the classical minimum $c_0 = 0$ we find

$$\begin{aligned} c_1 &= \frac{v^2}{2}(s_\alpha s_\beta^3 - c_\alpha c_\beta^3)\mathrm{Im}\bar{\lambda}_5 \\ &\quad + v^2(s_\alpha c_\beta \mathrm{Im}\bar{\lambda}_6 - c_\alpha s_\beta \mathrm{Im}\bar{\lambda}_7), \\ c_2 &= -\frac{v^2}{2}(s_\alpha c_\beta^3 + c_\alpha s_\beta^3)\mathrm{Im}\bar{\lambda}_5 \\ &\quad - v^2(c_\alpha c_\beta \mathrm{Im}\bar{\lambda}_6 + s_\alpha s_\beta \mathrm{Im}\bar{\lambda}_7). \end{aligned} \tag{44}$$

The second order terms hA and HA in (42) can be removed as usual by the orthogonal rotation a_{ij} $(i,j = 1,2,3)$ in h,H,A sector

$$(h,H,A)\; M^2 \begin{pmatrix} h \\ H \\ A \end{pmatrix} = (h_1,h_2,h_3)\; a_{ik}^{\rm T}\; M_{kl}^2\; a_{lj} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \tag{45}$$

where the mass matrix has the form

$$M^2 = \frac{1}{2}\begin{pmatrix} m_h^2 & 0 & c_1 \\ 0 & m_H^2 & c_2 \\ c_1 & c_2 & m_A^2 \end{pmatrix}. \tag{46}$$

The squared masses of the physical states h_1, h_2, h_3, which are the Higgs bosons without definite CP-parity, are defined by the eigenvalues of the mass matrix M^2 (the roots of the cubic equation for the eigenvalues are given by the Cardano formulae):

$$\begin{aligned} m_{h2}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\theta}{3}\right) - \frac{a_2}{3}, \\ m_{h1}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\theta+2\pi}{3}\right) - \frac{a_2}{3}, \\ m_{h3}^2 &= 2\sqrt{(-q)}\cos\left(\frac{\theta+4\pi}{3}\right) - \frac{a_2}{3}, \end{aligned} \tag{47}$$

where

$$\theta = \arccos\frac{r}{\sqrt{(-q^3)}},$$

$$r = \frac{1}{54}(9a_1a_2 - 27a_0 - 2a_2^3), \quad q = \frac{1}{9}(3a_1 - a_2^2),$$

[2] However, with non-zero λ_5, λ_6 and λ_7, the factor of the scalar–pseudoscalar Higgs counterterm is not explicitly proportional to the tadpole renormalization constant, or the tadpole parameter c_0

$$a_0 = c_1^2 m_H^2 + c_2^2 m_h^2 - m_h^2 m_H^2 m_A^2,$$
$$a_1 = m_h^2 m_H^2 + m_h^2 m_A^2 + m_H^2 m_A^2 - c_1^2 - c_2^2,$$
$$a_2 = -m_h^2 - m_H^2 - m_A^2.$$

One can see that in the limiting case of a CP-conserving potential $c_{1,2} \to 0$ the following correspondence occurs: $m_{h_1} \to m_h$, $m_{h_2} \to m_H$ and $m_{h_3} \to m_A$. The normalized eigenvectors of the matrix M^2, which are at the same time the matrix elements of a_{ij}, $(h, H, A) = a_{ij} h_j$, have the form $a_{ij} = a'_{ij}/n_j$, where

$$a'_{11} = ((m_H^2 - m_{h_1}^2)(m_A^2 - m_{h_1}^2) - c_2^2), \quad a'_{21} = c_1 c_2,$$
$$a'_{31} = -c_1(m_H^2 - m_{h_1}^2),$$
$$a'_{12} = c_1 c_2, \quad a'_{22} = ((m_h^2 - m_{h_2}^2)(m_A^2 - m_{h_2}^2) - c_1^2),$$
$$a'_{32} = -c_2(m_h^2 - m_{h_2}^2),$$
$$a'_{13} = -c_1(m_H^2 - m_{h_3}^2), \quad a'_{23} = -c_2(m_h^2 - m_{h_3}^2),$$
$$a'_{33} = (m_h^2 - m_{h_3}^2)(m_H^2 - m_{h_3}^2)$$

and $n_i = (a'^2_{1i} + a'^2_{2i} + a'^2_{3i})^{1/2}$ [3]. Representations for the triple and quartic Higgs boson self-interactions in the case of CP-violating potential are given by the expansions of the structures $a_{ij} h_j a_{ik} h_k a_{il} h_l$, and $a_{ij} h_j a_{ik} h_k a_{il} h_l a_{i,m} h_m$; they are bulky and not very telling, so we do not show them here. If the imaginary parts of $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are not small, large off-diagonal elements of the mixing matrix a_{ij} could appear, leading to significant mass splittings of the Higgs states and modifications of the Higgs boson interactions.

We assume that in the Yukawa sector $\langle\varphi_1\rangle$ couples only to down fermions:

$$\sum_{\alpha=d,s,b} V_{u\alpha} \frac{e m_\alpha}{2\sqrt{2} m_W s_W c_\beta} \times [\bar{\psi}_1 (1+\gamma_5) \psi_{2\alpha} \varphi_1 + \bar{\psi}_{2\alpha}(1-\gamma_5)\psi_1 \varphi_1^+], \tag{48}$$

where $(h, H, A) = a_{ij} h_j$, for the first generation quarks $\bar{\psi}_1 = \{\bar{u}, V_{ud}\bar{d} + V_{us}\bar{s} + V_{ub}\bar{b}\}$, $\psi_{2\alpha} = (d, s, b)$ and analogous terms for the c and t quarks (V_{ab} denotes the CKM matrix elements), and $\langle\varphi_2\rangle$ couples only to up fermions (model of type II [24]):

$$\frac{e m_u}{2\sqrt{2} m_W s_W s_\beta} \Big[\bar{\psi}_1 (1+\gamma_5) \mathrm{i}\tau_2 \psi_2 \varphi_2^+ + \bar{\psi}_2 (1-\gamma_5) \mathrm{i}\tau_2 \psi_1 \varphi_2\Big], \tag{49}$$

where again the physical h_1, h_2, h_3 states are introduced by means of the a_{ij} rotation, $\bar{\psi}_1 = \{\bar{u}, V_{ud}\bar{d} + V_{us}\bar{s} + V_{ub}\bar{b}\}$, $\psi_2 = u$ and analogous terms for the c and t quarks.

[3] No ordering of the masses $m_{h_1} < m_{h_2} < m_{h_3}$ is required. If we want to keep this ordering, then a_{ij} written above, valid for the case $m_H > m_A$, must be changed. For the case $m_H < m_A$ one should replace $m_{h_2} \leftrightarrow m_{h_3}$ and change the sign of a_{i2}, a_{i3} in the expressions for a_{ij}

4 Higgs–gauge boson and Higgs–fermion couplings in the MSSM with explicit CP-violation

In the following we shall focus on the MSSM scenario for the two-Higgs-doublet model, which allows us to restrict strongly the THDM parameter space. It is not the only one possible; standard model-like scenarios in the general two-Higgs-doublet model have been discussed in [25]. A detailed consideration in the framework of MSSM has been performed in [5] (see also [6]). In this section we would like only to compare qualitatively our results with the results of these approaches. Our calculation follows a somewhat different scheme. In [5] the tree-level two-Higgs-doublet potential is CP-invariant. The phase ξ of μ_{12}^2 is radiatively induced by the tadpole diagrams and can be absorbed in the definition of the μ parameter which appears in the stop mixing matrix off-diagonal element $A_t - \mu/\mathrm{tg}\beta$. The $\bar{\lambda}_5$-, $\bar{\lambda}_6$- and $\bar{\lambda}_7$-terms are also radiatively induced by the threshold effects. At the same time the trilinear couplings A_t, A_b also carry a phase[4], so both the radiatively induced and the trilinear phases contribute to the phase $\arg(\mu A)$ of the $\bar{\lambda}_5$-, $\bar{\lambda}_6$- and $\bar{\lambda}_7$-terms. We do not account for the radiatively induced phase which is calculated diagrammatically. In the case under consideration the fine-tuning conditions (38) and (39) are not fulfilled, so CP-invariance of the two-doublet potential is explicitly broken by complex parameters at the tree level. The real and imaginary parts of the parameter μ_{12}^2 are defined by means of the condition (41) for the real parts of the parameters $\bar{\lambda}_{5,6,7}$ and the minimization condition (43) (where $c_0 = 0$) for their imaginary parts. In the following calculations the complex parameters $\bar{\lambda}_{5,6,7}$ are specified in the framework of the MSSM.

We used the two-loop symbolic results for $\bar{\lambda}_i$, $i = 1, \cdots, 7$, which were obtained in the RG approach [17] and extended to the case of CP-violation in [5]. The parameters $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are non-zero in the next-to-leading order approximation (RG improved leading order approximation), so using (41) and (43)

$$\mathrm{Re}\mu_{12}^2 = m_A^2 s_\beta c_\beta + v^2 \left(s_\beta c_\beta \mathrm{Re}\bar{\lambda}_5 + \frac{1}{2} c_\beta^2 \mathrm{Re}\bar{\lambda}_6 + \frac{1}{2} s_\beta^2 \mathrm{Re}\bar{\lambda}_7 \right), \tag{50}$$

$$\mathrm{Im}\mu_{12}^2 = \frac{v^2}{2} (s_\beta c_\beta \mathrm{Im}\bar{\lambda}_5 + c_\beta^2 \mathrm{Im}\bar{\lambda}_6 + s_\beta^2 \mathrm{Im}\bar{\lambda}_7), \tag{51}$$

where $\bar{\lambda}_{5,6,7}$ depend on the finite term corrections to the leading logarithmic result which appear from the one-loop diagrams with trilinear couplings. The analytical representation of [5] has the form

$$\bar{\lambda}_5 = -\frac{3}{192\pi^2} h_t^4 \frac{\mu^2 A_t^2}{M_{\mathrm{SUSY}}^4} \left[1 - \frac{1}{16\pi^2}(2h_b^2 - 6h_t^2 + 16 g_s^2) t\right] - \frac{3}{192\pi^2} h_b^4 \frac{\mu^2 A_b^2}{M_{\mathrm{SUSY}}^4} \left[1 - \frac{1}{16\pi^2}(2h_t^2 - 6h_b^2 + 16 g_s^2) t\right],$$

[4] For a rewiev see e.g. [26]

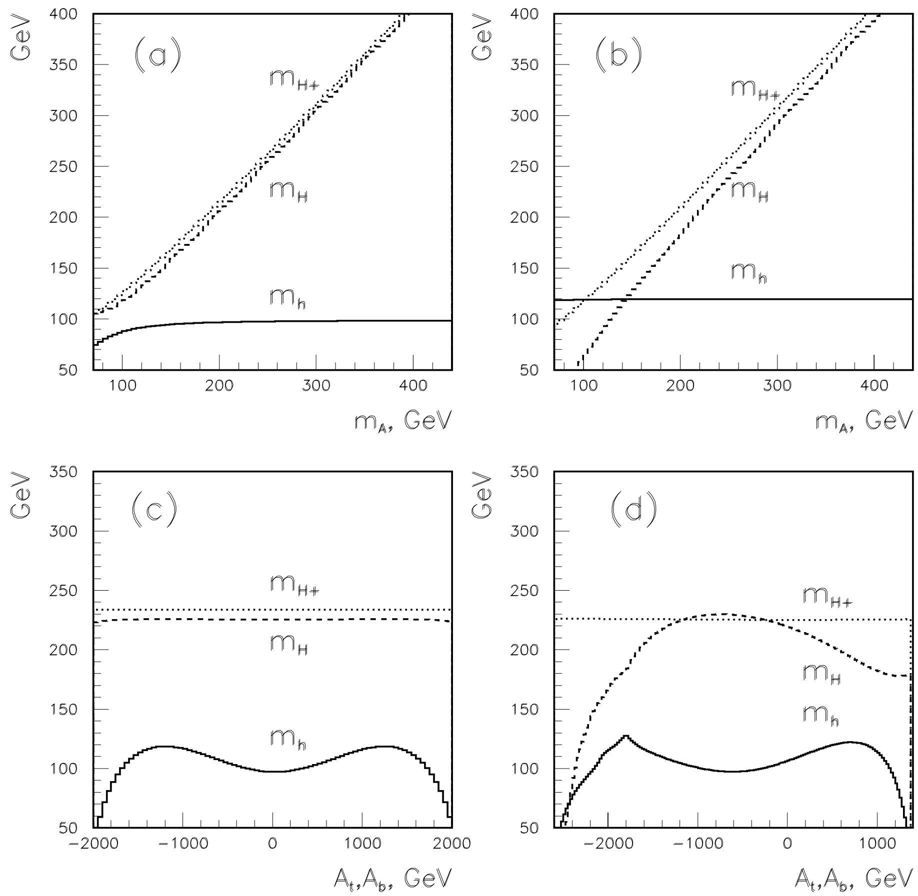

Fig. 1. Masses of the neutral and charged Higgs bosons $h, H, H^{\pm}$ versus the pseudoscalar mass m_A and the trilinear constants A_t, A_b calculated by means of (31)–(33) with the analytical $\bar{\lambda}_i$ ($i = 1, \cdots, 7$) parameterization of [5]. The $\Delta\bar{\lambda}_5$ is chosen to be positive. The CP-conserving limit $\theta = 0$ is taken. (a) tg$\beta = 4$, $M_{\rm SUSY} = 0.5$ TeV, $A_t = A_b = \mu = 0$; (b) tg$\beta = 4$, $M_{\rm SUSY} = 0.5$ TeV, $A_t = A_b = 0.9$ TeV, $\mu = -1.5$ TeV; (c) tg$\beta = 4$, $M_{\rm SUSY} = 0.5$ TeV, $m_A = 220$ GeV, $\mu = 0$, $A_t = A_b$; (d) tg$\beta = 4$, $M_{\rm SUSY} = 0.5$ TeV, $m_A = 220$ GeV, $\mu = -2$ TeV, $A_t = A_b$. Very small variations of the charged Higgs boson mass $m_{H^\pm}$ in (d) are due to the cancellation of the leading power terms $\sim \mu^2 A^2_{t,b}/M^4_{\rm SUSY}$, see [5], in the difference of $\Delta\bar{\lambda}_4$ and $\Delta\bar{\lambda}_5$; see (33). If $\Delta\bar{\lambda}_5$ is chosen to be negative (purely imaginary μA in (50)), m_H increases in comparison with the case $A_t = A_b = \mu = 0$

$$
\begin{aligned}
\bar{\lambda}_6 = &\frac{3}{96\pi^2} h_t^4 \frac{|\mu|^2 \mu A_t}{M^4_{\rm SUSY}} \\
&\times \left[1 - \frac{1}{16\pi^2}\left(\frac{7}{2}h_b^2 - \frac{15}{2}h_t^2 + 16 g_s^2\right) t\right] \\
&- \frac{3}{96\pi^2} h_b^4 \frac{\mu}{M_{\rm SUSY}} \left(\frac{6A_b}{M_{\rm SUSY}} - \frac{|A_b|^2 A_b}{M^3_{\rm SUSY}}\right) \\
&\times \left[1 - \frac{1}{16\pi^2}\left(\frac{1}{2}h_t^2 - \frac{9}{2}h_b^2 + 16 g_s^2\right) t\right], \\
\bar{\lambda}_7 = &\frac{3}{96\pi^2} h_b^4 \frac{|\mu|^2 \mu A_b}{M^4_{\rm SUSY}} \\
&\times \left[1 - \frac{1}{16\pi^2}\left(\frac{7}{2}h_t^2 - \frac{15}{2}h_b^2 + 16 g_s^2\right) t\right] \\
&- \frac{3}{96\pi^2} h_t^4 \frac{\mu}{M_{\rm SUSY}} \left(\frac{6A_t}{M_{\rm SUSY}} - \frac{|A_t|^2 A_t}{M^3_{\rm SUSY}}\right) \\
&\times \left[1 - \frac{1}{16\pi^2}\left(\frac{1}{2}h_b^2 - \frac{9}{2}h_t^2 + 16 g_s^2\right) t\right]. \qquad (52)
\end{aligned}
$$

$A_{t,b}$ and μ are the factors in front of Higgs–squark(left)–squark(right) trilinear terms, $M_{\rm SUSY}$ is the SUSY energy scale, $m_{\rm top}$, m_b are the on-shell running masses of the third generation quarks, and $t = \log(M^2_{\rm SUSY}/m^2_{\rm top})$, $h_t = (\sqrt{2}m_{\rm top})/(v s_\beta)$, $h_b = (\sqrt{2}m_b)/(v c_\beta)$, $g_s = (4\pi\alpha_s)^{1/2}$. The trilinear parameters A_t, A_b and μ of the Higgs boson interaction with the third generation squarks can be, generally speaking, complex. In this case the $\bar{\lambda}_i$, $i = 1, \cdots, 7$ parameters of the two-doublet-Higgs potential are defined by tgβ, the SUSY scale $M_{\rm SUSY}$, and six relevant parameters in the sector of the Higgs boson interaction with the third generation squarks: μ, arg(μ), A_t, arg(A_t), A_b, arg(A_b). In the following consideration for simplicity we assume $|A_t| = |A_b|$ and assign the universal phase θ to μA_t and μA_b so that $\theta = \arg(\mu A_t) = \arg(\mu A_b)$. Then using the explicit structure of (50), the CP-invariance conditions (39) can be rewritten in the form ${\rm Im}(\overset{*}{\mu}{}^2_{12}\, \mu A) = 0$ [5].

The couplings of W and Z bosons to the h_1, h_2, h_3 scalars have the form

$$
\begin{aligned}
&V_\mu\, V_\nu\, h_1 \qquad f_V g_{\mu\nu}(c_{\alpha-\beta} a_{21} - s_{\alpha-\beta} a_{11}), \\
&V_\mu\, V_\nu\, h_2 \qquad f_V g_{\mu\nu}(c_{\alpha-\beta} a_{22} - s_{\alpha-\beta} a_{12}), \\
&V_\mu\, V_\nu\, h_3 \qquad f_V g_{\mu\nu}(c_{\alpha-\beta} a_{23} - s_{\alpha-\beta} a_{13}),
\end{aligned}
$$

where $V = W, Z$, $f_W = (e/s_{\rm W}) m_W$ and $f_Z = (e/(s_{\rm W} c^2_W))$ m_W. The couplings of the h_1, h_2, h_3 bosons to the t and b quarks have the form

$$
\begin{aligned}
&\bar{t}\; t\; h_1 \qquad f_t \frac{1}{s_\beta}(s_\alpha a_{21} + c_\alpha a_{11} - {\rm i} c_\beta a_{31} \gamma_5), \\
&\bar{t}\; t\; h_2 \qquad f_t \frac{1}{s_\beta}(s_\alpha a_{22} + c_\alpha a_{12} - {\rm i} c_\beta a_{32} \gamma_5), \\
&\bar{t}\; t\; h_3 \qquad f_t \frac{1}{s_\beta}(s_\alpha a_{23} + c_\alpha a_{13} - {\rm i} c_\beta a_{33} \gamma_5),
\end{aligned}
$$

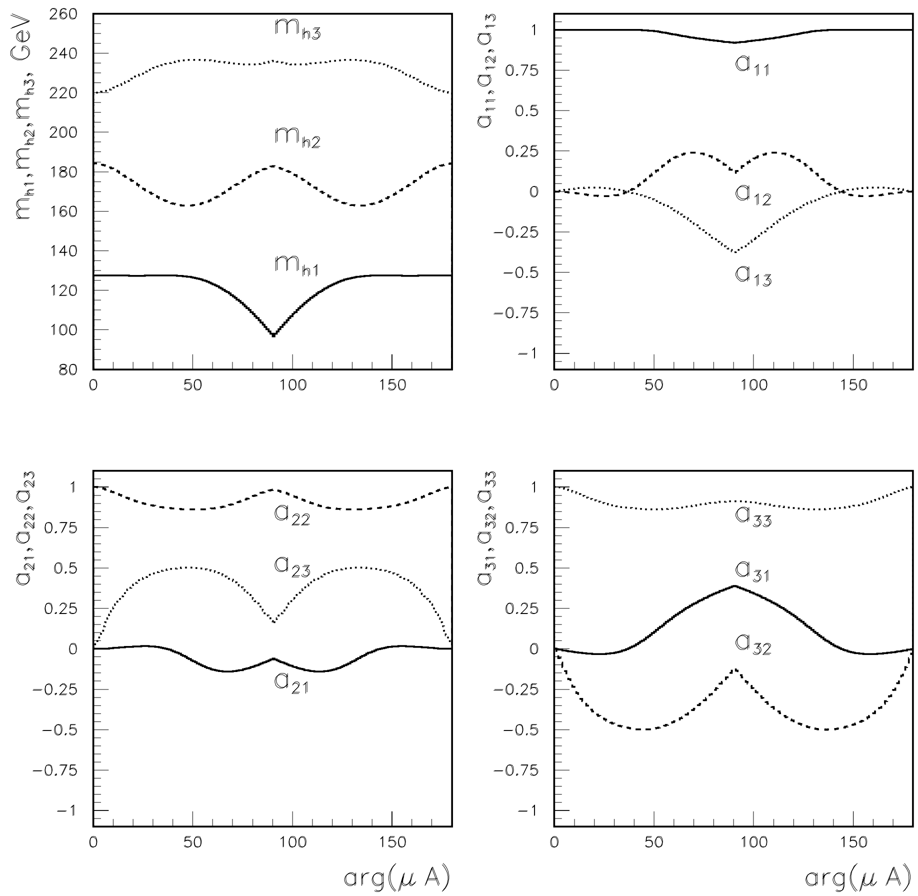

Fig. 2. Masses of the neutral Higgs bosons and the mixing matrix elements as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ parameters are taken from [5] at the MSSM parameter values $\mathrm{tg}\beta = 4$, $m_A = 220$ GeV, $M_{\mathrm{SUSY}} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV

$$\bar{b}\, b\, h_1 \qquad f_b \frac{1}{c_\beta}(c_\alpha a_{21} - s_\alpha a_{11} - \mathrm{i} s_\beta a_{31}\gamma_5),$$

$$\bar{b}\, b\, h_2 \qquad f_b \frac{1}{c_\beta}(c_\alpha a_{22} - s_\alpha a_{12} - \mathrm{i} s_\beta a_{32}\gamma_5),$$

$$\bar{b}\, b\, h_3 \qquad f_b \frac{1}{c_\beta}(c_\alpha a_{23} - s_\alpha a_{13} - \mathrm{i} s_\beta a_{33}\gamma_5),$$

where $f_{t,b} = (-e/(2s_{\mathrm{W}}))(m_{t,b}/m_W)$.

The Higgs boson mass spectrum of the CP-conserving limit $\theta = 0$ (in this limit $a_{ij} = \mathrm{diag}\{1,1,1\}$) is shown in Fig. 1. For the case of explicit CP-violation in the two-doublet-Higgs potential we take the parameter set $\mu = -2\,\mathrm{TeV}$, $A_t = A_b = -1.8\,\mathrm{TeV}$, $M_{\mathrm{SUSY}} = 0.5\,\mathrm{TeV}$, $m_A = 220\,\mathrm{GeV}$, $\mathrm{tg}\beta = 4$, which is typical for the region of MSSM parameter space where the imaginary parts of $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are large (of the order of 0.1–1) [5]. We demonstrate in Fig. 2 the neutral Higgs boson masses given by (47) and the mixing matrix elements a_{ij} as a function of the universal phase $\theta = \arg(\mu A_{t,b})$. The Higgs boson masses of the CP-conserving limit are substantially changed when the phase θ is not small. The m_{h1} in Fig. 2 is always smaller than m_h, and m_{h_2} has a downfall at the phase values around $\pi/4$. The Higgs-vector boson WWh_i, ZZh_i and the Higgs–fermion $q\bar{q}h_i$ ($q = t, b$) interaction vertices as a function of the phase θ are shown in Fig. 3. One can observe that the h_1 couplings to the gauge bosons W, Z decrease by about 15% if the phase of $\bar{\lambda}_5$, $\bar{\lambda}_6$, $\bar{\lambda}_7$ is large enough. Non-zero couplings of h_3 to the gauge bosons appear. The changes of the $b\bar{b}h_1$ and the $b\bar{b}h_2$ coupling regime are also rather pronounced (see Fig. 3). In the region of MSSM parameter space where the m_{h_3} is around 150–250 GeV and the μ and $A_{t,b}$ parameters are of the order of TeV, we have the regime of strong mixing in the Higgs sector. As a result the light Higgs boson h_1 could have not been observed at LEP2 ($\sqrt{s} = 200\,\mathrm{GeV}$) in the production channels $e^+e^- \to h_1 Z$, $e^+e^- \to \nu_e\bar{\nu}_e h_1$ because of the suppressed couplings to the gauge bosons, while the h_2, h_3 bosons are sufficiently heavy to be not produced on mass-shell at the LEP2 energy. A detailed analysis of this scenario can be found in [5,6].

[5] A detailed discussion of possible combined constraints on the MSSM parameter space from cosmology, direct searches and indirect measurements (rare decays) can be found in [27]

5 Triple and quartic Higgs boson couplings in the MSSM with explicit CP-violation

In the regions of the MSSM parameter space where the couplings of the lightest Higgs boson h_1 to the gauge bosons and top quark are suppressed, the traditional channels of Higgs boson production by radiation from a W, Z or t line and WW, ZZ fusion can have too small a rate to

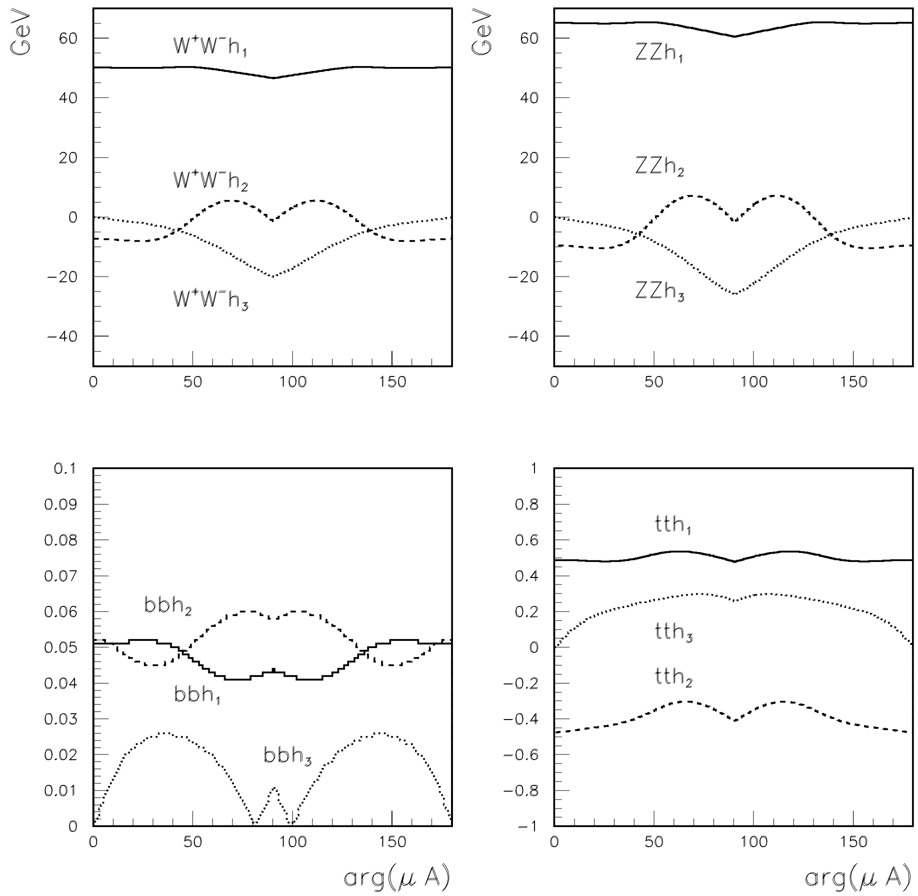

Fig. 3. Higgs–gauge boson and Higgs–fermion vertex factors as a function of the $\bar\lambda_6$ and $\bar\lambda_7$ phase. The $\bar\lambda_i$ parameters are taken from [5] at the MSSM parameter values tg$\beta = 4$, $m_A = 220$ GeV, $M_{\rm SUSY} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV. For the coupling with fermions we plot $({\rm Im}^2 g_{ffh} + {\rm Re}^2 g_{ffh})^{1/2}$

be experimentally observed. For this reason it is interesting to consider the possibility of double Higgs production (like $gg \to h_2 \to h_1 h_1$) defined by the self-coupling vertices. Such calculations are known in the CP-conserving limit [8], when the cross sections of double and triple Higgs boson production turn out to be very small. Only some of them are accessible for observation at high luminosity colliders. In the case of CP-violation some self-couplings may be substantially increased, providing better opportunities for the experimental reconstruction.

The $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ potential terms can modify significantly the Higgs boson self-interaction vertices calculated in the leading one-loop approximation with $\bar\lambda_i = 0$ ($i = 5, 6, 7$). At the next-to-leading order approximation the $\bar\lambda_i$ couplings (50) include terms of the order of $h^4_{t,b}\mu^2 A^2_{t,b}/M^4_{\rm SUSY}$ and $h^4_{t,b}\mu A_{t,b}/M^2_{\rm SUSY}$, so they can reach values of the order of 0.1–1 at moderate values of $M_{\rm SUSY}$ and μ and $A_{t,b}$ taken at the TeV energy scale. For example, in the CP-conserving limit $\theta = 0$ the hhh vertex in the mass parameterization has the form

$$g_{hhh} = \frac{3e}{m_W s_{\rm W} s_{2\beta}}[-(c_\beta c^3_\alpha - s_\beta s^3_\alpha)m^2_h + c^2_{\beta-\alpha} c_{\beta+\alpha} m^2_A + c^2_{\beta-\alpha}(\bar\lambda_5 c_{\beta+\alpha} + \bar\lambda_6 c_\beta s_\alpha - \bar\lambda_7 s_\beta c_\alpha)v^2]. \quad (53)$$

The contributions of the $\bar\lambda_{5,6,7}$-terms and the $m^2_{h,A}$-terms in this expression are of the same order if $\lambda_{5,6,7} \sim O(1)$. The rotation of the $\theta = 0$ mass eigenstates by the matrix a_{ij} defined by (45) gives the $g_{h_1h_1h_1}$ vertex a different form but also it has substantial contributions of the $\bar\lambda_{5,6,7}$-terms.

Using the parameter set described in the previous section, we show the values of various triple and quartic Higgs boson self-interaction vertices as a function of the universal phase $\theta = \arg(\mu A_t) = \arg(\mu A_b)$ in Fig. 4. The values of the Higgs boson self-interaction vertices in the CP-conserving limit $\theta = 0,\pi$ and in the leading order approximation $\bar\lambda_5 = \bar\lambda_6 = \bar\lambda_7 = 0$ are marked in Fig. 4 by horizontal arrows. The $\bar\lambda_5$, $\bar\lambda_6$ and $\bar\lambda_7$ potential terms induced in the next-to-leading order approximation introduce very large corrections to the triple and quartic self-interactions of Higgs bosons. In the region of the MSSM parameter space under consideration the difference of the leading order and the next-to-leading order vertex factors can be several times in some ranges of the phase variation.

6 Summary

We demonstrated the tree-level equivalence of the two-Higgs-doublet model potentials (1) and (9), where CP-invariance may be explicitly broken by the λ_6-term in (1) or by the complex μ^2_{12}-, $\bar\lambda_5$-terms in (9). The parameters λ_i ($i = 1, \cdots, 6$) of (1) and μ^2_1, μ^2_2, μ^2_{12}, $\bar\lambda_i$ ($i = 1, \cdots, 5$) of (9) are related by (10). In the case of real parameters the diagonalization of the potential (9) in the ground state

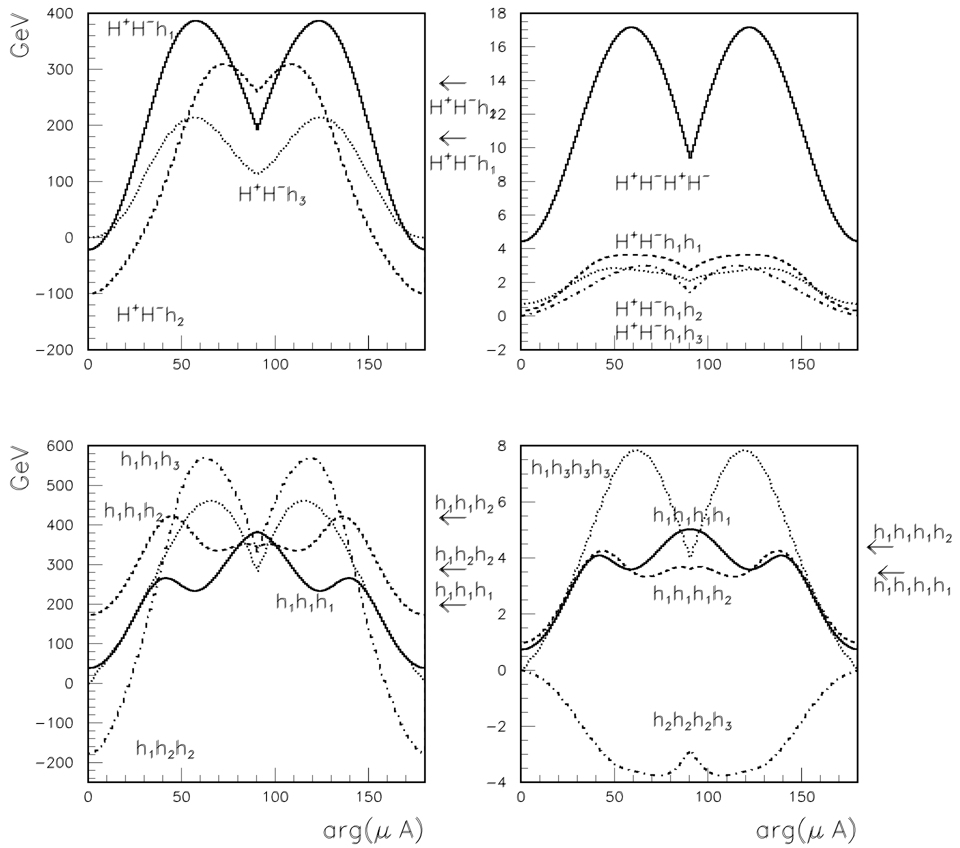

Fig. 4. Triple and quartic Higgs boson vertex factors as a function of the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ phase. The $\bar{\lambda}_i$ parameters are taken from [5] at the MSSM parameter values tg$\beta = 4$, $m_A = 220$ GeV, $M_{\rm SUSY} = 0.5$ TeV, $A_t = A_b = -1.8$ TeV, $\mu = -2$ TeV. Horizontal arrows indicate the values of vertex factors in the CP-conserving limit $\theta = 0$ and the leading order approximation $\bar{\lambda}_5 = \bar{\lambda}_6 = \bar{\lambda}_7 = 0$

can be performed by means of the substitutions (14)–(20) which express the $\bar{\lambda}_i$ and μ_1^2, μ_2^2 parameters through the Higgs boson masses $m_h, m_H, m_A, m_{H\pm}$, the mixing angles α, β and the μ_{12}^2 parameter. In the general case the $\bar{\lambda}_6$ and $\bar{\lambda}_7$ potential terms (21) should also be considered with the diagonalization and minimization conditions (22)–(28). If the complex parameters μ_1^2, μ_2^2, $\bar{\lambda}_i$ $(i = 1, \cdots, 7)$ and μ_{12}^2 are introduced, CP-invariance of the hermitian potential (37) is explicitly violated at the tree level unless the fine-tuning conditions (38) or (39) for the parameters are satisfied. So in the following we consider the problem of diagonalization in the local minimum for the two-Higgs-doublet potential which is not CP-invariant. For the diagonalization of the potential (37) again we use the substitution (22)–(28) to be taken for real parts of parameters. The minimization of potential (37) at the tree level occurs with the condition $c_0 = 0$ (43) for the imaginary parts of parameters. The imaginary parts of $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ give rise to the CP-odd/CP-even Higgs boson off-diagonal terms, which are removed by the orthogonal rotation in (h, H, A) space, giving mass eigenstates h_1, h_2, h_3 without definite CP-parity and with the mass spectrum and couplings substantially different from the masses and couplings of the CP-even and CP-odd states h, H, A, if the imaginary parts of the parameters $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ are sufficiently large.

In the framework of MSSM the real parts of the $\bar{\lambda}_i$ $(i = 1, \cdots, 5)$ parameters are fixed at the SUSY energy scale by the conditions (29). Radiative corrections to the $\bar{\lambda}_i^{\rm SUSY}$ $(i = 1, \cdots, 7)$ parameters are generated at the m_W energy scale. Equations (31)–(33) express the mixing angle α and the masses of the Higgs bosons in terms of the radiative corrections to $\bar{\lambda}_i^{\rm SUSY}$ $(i = 1, \cdots, 7)$ couplings (e.g. given by the RG evolution). They are valid independently on the particular scheme which is used for the calculation of the radiative corrections to the $\bar{\lambda}_i^{\rm SUSY}$ $(i = 1, \cdots, 7)$.

In the next-to-leading order approximation the complex $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ parameters are generated by the soft CP-violating Yukawa interactions of Higgs bosons with the scalar quarks. Using the results of [5] we calculated the Higgs–gauge boson, Higgs–fermion and the Higgs triple and quartic couplings for a representative MSSM parameter set, when the off-diagonal elements of the Higgs boson mixing matrix are large. The $\bar{\lambda}_5$, $\bar{\lambda}_6$ and $\bar{\lambda}_7$ parameters introduce significant corrections to the Higgs self-interaction, even in the case when their effects on the Higgs–gauge boson and Higgs–fermion couplings are rather small. These corrections could rather strongly (by one–two orders of magnitude in comparison with the case of CP-conservation) enhance or suppress some channels of multiple Higgs boson production at future colliders, providing discriminative tests of CP-violation in the Higgs sector and improved feasibility to reconstruct experimentally the Higgs potential.

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